Logic for Fun and Profit
A Preview

Chapter 1 – Mental Calisthenics

Puzzles

Puzzle-solving is a good mental warm-up for a logic class. The same kind of reasoning that is involved in solving a puzzle is also used in solving logic problems. The first step in solving a puzzle is to decide how the problem should be represented. If you can find a good way to represent the problem, you are well on your way to solving it. To understand the importance of representation, try to imagine how difficult it would be to do arithmetic using Roman numerals. Multiplying 25 by 13 is trivial, but try multiplying XXV by XIII (without converting).

Position Puzzles

In solving these puzzles, you need to represent them spatially. Once you have done so, the solution is fairly trivial.

> Three guys and three girls have been invited to a crawfish boil at Boudreaux's house. They all want to sit on the same side of one of Boudreaux's long tables (which seats six people on one side). No two people of the same gender want to sit next to each other. Tom does not want to sit by Jane. Dick is on one side of Jane and Harry is on the other side. April is in the first seat and Harry in the last one. Where does Carol sit?

Relationship Puzzles

There are different sorts of relationships involved in these puzzles. Some are comparisons among people based on factors such as height or age. Others are kinship relations.

> The contestants in the annual Bayou Teche swim meet were the alligator, the nutria, the duck, and the turtle. The nutria finished two places ahead of the duck. The alligator was neither first nor last and the nutria was not second. Who finished first and who finished last?

Matrix Puzzles

I call these matrix puzzles because it is useful to use a row-and-column or matrix format in representing the information that is given. This permits you to quite literally see how to solve the puzzle. This one is done as an example.

> The employees of a small loan company are Mr. Black, Mr. White, Mrs. Coffee, Miss Ambrose, Mr. Kelly, and Miss Earnshaw. The positions they occupy are manager, assistant manager, cashier, stenographer, teller, and clerk, though not necessarily in that order. The assistant manager is the manager's grandson, the cashier is the stenographer's son-in-law, Mr. Black is a bachelor, Mr. White is twenty-two years old, Miss Ambrose is the teller's step-sister, and Mr. Kelly is the manager's neighbor. Who holds each position? (Note: The solution of this puzzle is my own, but the puzzle came from another logic book. I would like to acknowledge the author, but unfortunately I have lost the reference.)

Below is the matrix before any information is recorded. Column headings represent the employees and row headings represent the positions.

	MissA	Mr. B	Mrs.C	MissE	Mr. K	Mr. W
Manager						
Asst.Mgr.						
Cashier						

Stenog.						
Teller						
Clerk						

To solve this puzzle, we have to assume that everyone is "proper", that is, no children out of wedlock, no incestuous relationships, etc. Let's take it one bit of information at a time, and gradually fill in the matrix.

The assistant manager is the manager's grandson.

- This means that the assistant manager is male, so we can eliminate all the women.
- Also, the manager must be married (or have been), so we can eliminate all the single women.

	MissA	Mr. B	Mrs.C	MissE	Mr. K	Mr. W
Manager	X			X		
Asst.Mgr.	X		X	X		
Cashier						
Stenog.						
Teller						
Clerk						

The cashier is the stenographer's son-in-law.

- The cashier must be male, so we can eliminate all the women.
- Also, the stenographer must be married (or have been), so we can eliminate all the single women.

	MissA	Mr. B	Mrs.C	MissE	Mr. K	Mr. W
Manager	X			X		
Asst.Mgr.	X		X	X		
Cashier	X		X	X		
Stenog.	X			X		
Teller						
Clerk						

Mr. Black is a bachelor.

- He cannot be the manager or stenographer, because they are married.
- He cannot be anyone's son-in-law, so he is not the cashier.

	MissA	Mr. B	Mrs.C	MissE	Mr. K	Mr. W
Manager	X	X		X		
Asst.Mgr.	X		X	X		
Cashier	X	X	X	X		
Stenog.	X	X		X		
Teller						
Clerk						

Mr. White is twenty-two years old.

- He’s not old enough to be a grandfather, so he cannot be the manager.
- He’s not old enough to have a son-in-law, so he cannot be the stenographer.

	MissA	Mr. B	Mrs.C	MissE	Mr. K	Mr. W
Manager	X	X		X		X
Asst.Mgr.	X		X	X		
Cashier	X	X	X	X		
Stenog.	X	X		X		X
Teller						
Clerk						

Miss Ambrose is the teller’s step-sister.

- Miss Ambrose is not the teller.
- Miss Ambrose must be the clerk (because there is no other position left for her).
- If Miss Ambrose is the clerk, then we can X out “Clerk” for everyone else.
- Thus, Miss Earnshaw must be the teller (because there is no other position left for her).
- If Miss Earnshaw is the teller, then we can X out “Teller” for everyone else.
- Thus, Mr. Black must be the assistant manager (because there is no other position left for him).
- If Mr. Black is the assistant manager, then we can X out “Asst.Mgr.” for everyone else.
- Thus, Mr. White must be the cashier (because there is no other position left for him).
- If Mr. White is the cashier, then Mr. Kelly cannot be.

	MissA	Mr. B	Mrs.C	MissE	Mr. K	Mr. W
Manager	X	X		X		X
Asst.Mgr.	X	***	X	X	X	X

Cashier	X	X	X	X	X	***
Stenog.	X	X		X		X
Teller	X	X	X	***	X	X
Clerk	***	X	X	X	X	X

Mr. Kelly is the manager's neighbor.

- Mr. Kelly cannot be the manager.
- Thus, Mr. Kelly must be the stenographer (because there is no other position left for him).
- Thus, Mrs. Coffee must be the manager (because there is no other position left for her).

	MissA	Mr. B	Mrs.C	MissE	Mr. K	Mr. W
Manager	X	X	***	X	X	X
Asst.Mgr.	X	***	X	X	X	X
Cashier	X	X	X	X	X	***
Stenog.	X	X	X	X	***	X
Teller	X	X	X	***	X	X
Clerk	***	X	X	X	X	X

Chapter 2 – Logic and Language

Examples of Mixed Metaphors

- The sacred cows have come home to roost.
- She'll get it by hook or ladder.

Examples of Misplaced Modifiers

- Calf born to farmer with two heads!
- The patient was referred to a psychiatrist with a severe emotional problem.
- A 30-year-old man was found murdered by his parents in his home late Saturday night.

Examples of Unintended Double Meaning

- The sermon this morning: "Jesus Walks on the Water". The sermon tonight "Searching for Jesus".
- A bean supper will be held on Tuesday evening in the church hall. Music will follow.
- Ladies Bible Study will be held Thursday morning at 10. All ladies are invited to lunch in the Fellowship Hall after the BS is done.

Malapropisms

Malapropisms frequently occur when people are trying to use words that are beyond their ordinary vocabulary. They sometimes occur in writing when a person uses a word or phrase that sounds right, but has a very different meaning. My favorite example is from an ethics class. I had asked the students to write a brief essay on euthanasia. One girl turned in an essay entitled Youth in Asia.

Some humorous examples of malapropisms occur in children's misunderstanding of adult conversation. I recall reading about one little girl who had visited a friend's church. When she got home, she told her mother that they sang a song about a bear named Gladys who didn't see well. When her mother inquired, she found out that the name of the hymn was *Gladly the Cross I Bear*, but the little girl had understood it as *Gladys the Cross-Eyed Bear.*

Sometimes humorists will deliberately create examples of malapropisms based on the hearing difficulties associated with old age. I recall seeing a cartoon showing three old men sitting on a park bench. The first one said, "It sure is windy today." The second one replied, "No, it's not Wednesday; it's Thursday," to which the last one responded, "I'm thirsty, too. Let's grab a beer."

Chapter 4 – Informal Logic

One of the annoying things about mathematicians and scientists is that they use ordinary words in very un-ordinary ways. Rather than inventing a new word, they will give an everyday word a technical meaning. Thus, the word "function" is defined by mathematicians to mean "a magnitude so related to another magnitude that to values of the latter there correspond values of the former" (whatever that means). This specialized use of everyday words can sometimes be confusing to the rest of us who are not part of the secret brotherhood (or whatever) that created the peculiar usage.

Unfortunately, logicians also play this game. In ordinary speech, the word "formal" refers to the way you would dress for tea with the Queen, whereas "informal" designates the attire more appropriate for a crawfish boil in the back yard (normally jeans and tee shirt). But logicians use these two words to refer to two different types of errors in reasoning (known as fallacies). A formal fallacy is an error that depends on the form or structure of an argument, without regard to its content. As we shall see later, formal fallacies can be represented using mathematical symbolism, and the errors can be demonstrated using techniques similar to those used to demonstrate errors in algebra or trigonometry.

By contrast, informal fallacies are dependent upon the content of the argument, rather than its form, and on the particular way in which the argument is worded. Some logicians prefer not to include informal fallacies in logic texts, because they regard them as misuses of language rather than errors in reasoning. However, I think this distinction is rather arbitrary since our reasoning is normally done within the context of language usage. Moreover, these errors are so common, and so effective at misleading people, that I continue to recommend their study. It will not be safe to confine them to the trash bins of history until people cease to be seduced by them. I think that time is far away.

Some examples of informal fallacies:

- Amphiboly: the fallacy is due to an ambiguous grammatical structure. Example: "When my son Michael was 6 years old, he loved to wrestle with me before his nightly bedtime story. Once, when I came home sick and went to bed early, my wife intercepted Michael on his way to my bedroom and explained that I wasn't feeling well and wouldn't want to wrestle. 'Oh, yes, he does,' he replied; 'because I asked Daddy if he wanted to wrestle, and he said it was the last thing he wanted to do today.'" In this example, the father is using a figure of speech to indicate that he does not want to wrestle at all, but his son thinks he means that he wants wrestling to be his final activity of the day.

- Appeal to pity: substituting emotion for reason so that people will agree with you because they feel sorry for you. Example: "Prof. Meany, I think you should consider raising my grade. I really need to keep my average up so I can get in the nursing program. My husband left me and I have 3 kids who depend on me for their sole support." As a principle of distributive justice, need is a relevant criterion in deciding who should receive food stamps, public housing, and other social welfare services. However, assignment of grades is not part of the social welfare system, so it doesn't matter how badly a student needs a good grade.
- Utter irrelevance (*non sequitur*): the premises have nothing to do with the conclusion. Former President Reagan was known for this. On one occasion he observed (correctly) that "Veterans have always had a strong voice in our government." Then he added the non sequitur: "It's time to give them the recognition they so rightly deserve." Arguments like this are so dumb that no one should be deceived by them. However, I have learned over the years never to underestimate the depths of human stupidity. As Albert Einstein said, "Two things are infinite, the universe and human stupidity, and I'm not sure about the universe."
- Begging the question (*petitio principii* or circular argument): assuming as true what you are trying to prove. Example: In a Miss Universe contest several years ago, one of the contestants was asked, "If you could live forever, would you? And why?" She answered, "I would not live forever, because we should not live forever, because if we were supposed to live forever, then we would live forever, but we cannot live forever, which is why I would not live forever."
- False dilemma: trying to convince your audience that they must choose between two extreme views when there are actually intermediate positions. Example: "America – love it or leave it!" The implied message is that if you don't like America the way it is, you should leave, because discontent shows lack of patriotism. But you could love your country dearly and still want to change it.

Chapter 5 – Formal Logic

There are four types of categorical propositions, conventionally designated as follows:

- Type A: universal affirmative ("All ... are").
- Type E: universal negative ("No ... are").
- Type I: particular affirmative ("Some ... are").
- Type O: particular negative ("Some ... are not").

To call attention to the form of an argument, as opposed to its content, it will be useful to symbolize the propositions. By convention, capital letters are used to designate the subject and predicate terms, with one of the type letters (a, e, i, or o – but not u) between the letters for the two terms. Using the examples above:

"All dogs are mammals." would be symbolized as DaM.
"No bachelors are married men." would be symbolized as BeM.

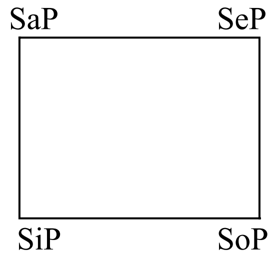
Immediate Inference

The term "immediate inference" refers to what can be inferred from a single proposition, without the addition of any other information. In effect, we have an argument with a single premise.

By way of introduction to this topic, let's think about what we mean when we say that two statements are opposite in meaning. Our ordinary use of the word "opposite" is not very precise. We can make our meaning more precise by a series of definitions. For the present, we will restrict our discussion to categorical propositions that are in standard form.

- Such propositions can be opposite in quality, that is, either affirmative or negative.
- They can be opposite in quantity, that is, either universal or particular.
- They can be opposite in truth-value, that is, either true or false.

If we use S to represent the subject term and P to represent the predicate term, we can show the relationships among propositions having the same subject and predicate terms by means of the following diagram. This diagram has traditionally been called the *Square of Opposition*, although some of my students dubbed it the Square of Confusion.



The propositions on opposite corners of the square (diagonally) are called *contradictories*. They are completely opposite, that is, they are opposite in quantity, quality, and truth-value. That means we can infer the falsity of one from the truth of the other, or vice versa.

Example: "All dogs are mammals" is true.
Therefore "Some dogs are not mammals" is false.
Example: "No dogs are mammals" is false.
Therefore "Some dogs are mammals" (meaning "at least some") is true.

The propositions across the square from each other at the top and the bottom are not completely opposite. They are opposite in quality but not in quantity. First, consider the two propositions at the top: SaP and SeP. They are called *contraries*. They cannot both be true, but they could both be false. So if we know that SaP is true, we can infer that SeP is false.

Example: "All dogs are mammals" is true.
Therefore "No dogs are mammals" is false.

If, however, SaP is false, the truth-value of SeP cannot be determined.

Example: "All dogs are cats" is false.
But "No dogs are cats" cannot be inferred.
Because: "All dogs are black" is false.
But "No dogs are black" cannot be inferred.

As the previous example shows, sometimes both contraries are false, sometimes not. That is why no inference can be made if all we know is that one of them is false.

The propositions at the bottom of the square are called *sub-contraries*. They cannot both be false, but they could both be true. So if we know that SiP is false, we can infer that SoP is true.

Example: "Some dogs are cats" is false.
Therefore "Some dogs are not cats" (meaning "at least some") is true.

If, however, SiP is true, the truth-value of SoP cannot be determined

Example: "Some dogs are black" is true.
But "Some dogs are not black" cannot be inferred.
Because: "Some dogs are mammals" (meaning "at least some") is true.
But "Some dogs are not mammals" cannot be inferred.

As the previous example shows, sometimes both sub-contraries are true, sometimes not. That is why no inference can be made if all we know is that one of them is true.

Chapter 7 – Mathematical Logic

In mathematics, an operation such as addition is indicated by the expression 3+5. In that expression, + is an operator, and the numbers 3 and 5 are called operands. If we replace the numbers in a numeric expression with variables – such as X+Y – it would be clear that the letters represented numbers.

If we construct an expression containing more than one operator, we can have ambiguity of meaning. For example, the expression 3+5x2 could yield either 16 or 13, depending on whether the addition or the multiplication is done first. One way to resolve the ambiguity is to use parentheses to show which operation should be performed first: (3+5) x 2 = 8x2 = 16

3 + (5x2) = 3+10 = 13

Logical Operators

We can represent logical operations by the use of logical operators. For example, the operation of negation is represented by NOT. In numeric operations the operands are numbers, and variables can be used to represent numbers. In logical operations the operands are truth-values, and variables can be used to represent statements that can be either true or false. Thus, if A is a statement, the negation of that statement would be represented by NOT A.

Besides negation, the fundamental logical operations are conjunction and disjunction, represented by the logical operators AND and OR. It makes sense to negate a single statement, but you can't conjoin (or disjoin) one thing. In other words, the operations of conjunction and disjunction require two operands: A AND B or A OR B.

If we wish to construct complex expressions, such as A AND B OR C we will use parentheses to avoid ambiguity: (A AND B) OR C or A AND (B OR C)

Truth Tables

Truth tables can be used to show all possible truth-values for an expression. For example, there are only two possible values for the expression NOT A.

A	NOT A
T	F
F	T

With two-operand expressions, there are four possible combinations of truth-values:

A	B	A AND B
T	T	T
T	F	F
F	T	F
F	F	F

Truth tables can also be regarded as defining how an operator is to be used. For example, the word "or" is ambiguous in English. It can mean either/or (exclusive) or and/or (inclusive). To avoid ambiguity, the logical OR is given the inclusive meaning:

A	B	A OR B
T	T	T
T	F	T
F	T	T
F	F	F

If we wished to define an operator for the exclusive use of "or" (XOR, as it is sometimes called), we can construct a complex expression. In ordinary English, we say that either/or means one or the other but not both. The corresponding logical expression is (A OR B) AND NOT (A AND B). The truth table would look like this:

A	B	A OR B	A AND B	NOT(A AND B)	(A OR B) AND NOT(A AND B)
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

The truth table verifies our everyday understanding of the exclusive "or"; that is, such a statement is true only when the constituent statements have different truth-values. If it's truly an either/or situation, they can't both be true at the same time. You can vote for a Democrat or a Republican for President, but not both. (Perhaps it is more accurate to say that you cannot legally vote for both. However, I am reminded of the old Louisiana election-day maxim: "Vote early and vote often!").

We have seen that a truth table with only one variable has two rows, while one with two variables requires four rows. If we add a third variable, the number of rows necessary to show all possible combination of truth-values would be eight. In general, the number of rows in a truth table is 2^N, where N is the number of variables.

Chapter 8 – Computer Logic

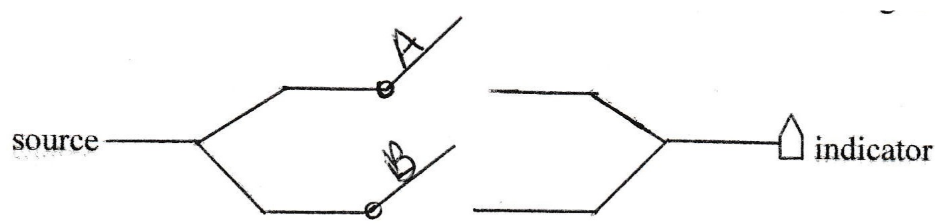
Circuit Design

To be able to perform logical operations, a computer has logic circuits built into its circuit board. The fundamental logic circuits implement the logical operators NOT, AND, and OR.

A simplified form of the AND circuit looks something like this for the expression A AND B:

This assumes a flow of electricity from the source to the light (or some other indicator). To complete the circuit, both the A and B gates must be closed.

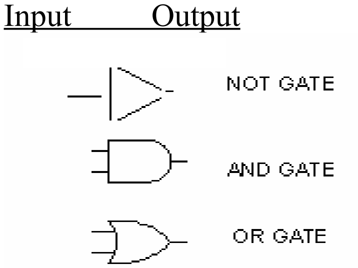
Before drawing the OR circuit, we need to discuss the ambiguity of the word *or* in English: it can mean either/or (exclusive) or and/or (inclusive). By definition, the OR operator is defined as the inclusive *or*. The circuit for the expression A OR B would look something like this:



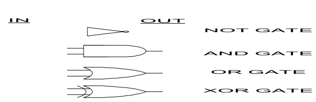
To complete this circuit, either the A gate or the B gate, or both, may be closed.

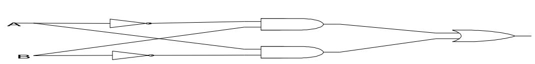
The basic circuits are called gates and are symbolized as follows:

<u>Input</u> <u>Output</u>

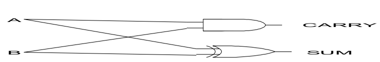


Now let us return to the OR operator. It is defined as the inclusive *or*. If we wish to use an exclusive *or* (called XOR), we require a derived circuit. What is meant by A XOR B is that A and B cannot both be true. (A AND NOT B) OR (NOT A AND B)



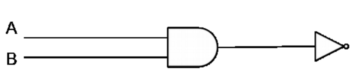


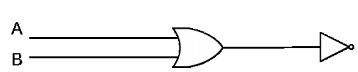
CIRCUIT FOR XOR GATE



CIRCUIT FOR ADDER

The terms NAND and NOR are obviously short-hand for NOT AND and NOT OR. Here are their circuits:





Chapter 12 – Inductive Inference

Mill's Methods

In the 19th century, the logician John Stuart Mill developed several techniques for justifying a statement that A causes B.

The Method of Agreement: If two or more instances of the phenomenon under investigation have only one circumstance in common, then this common circumstance is the cause (or effect) of the phenomenon under investigation.

> Example: Suppose several friends have lunch together at a local restaurant. Within an hour, three of them get sick at the stomach, but the others feel fine. Using the method of agreement, we attempt to determine whether the three who got sick ate the same food, and the others did not. Upon investigation, we find that the three who got sick ate a very spicy *sauce piquante*, which the others did not have. We conclude that their stomach ailments were probably caused by the *sauce piquante*.

The Method of Difference: Suppose you are trying to determine the cause of a given phenomenon. If there are two sets of circumstances that are similar in all respects except one, and in one the phenomenon under investigation occurs and in the other it does not occur, the circumstance in which alone the two instances differ, is the effect, or the cause, or an indispensable part of the cause, of the phenomenon under investigation.

> Suppose, in the preceding example, we discover that one person – Joe – who ate the *sauce piquante* did not get sick. We might be inclined at first to think it was not the *sauce piquante* that made the others sick. But before jumping to that conclusion, we inquire whether there is anything different about Joe that would explain why he didn't get sick. Upon investigation, we find that Joe has a reputation for a cast-iron stomach: he can eat almost anything and not get sick. So we have used both the method of agreement and the method of difference to explain why most of the people who are the *sauce piquante* got sick, but one did not.

The Joint Method of Agreement and Difference: This method consists in the use of both the Method of Agreement and the Method of Difference in the same investigation.

> Our first example was really the joint method, because we found that those who got sick consumed the same food (agreement), and those who did not get sick consumed different food (difference). It seemed at first that we had found an exception with Joe, until we found another explanation for the different result in his case.

The Method of Concomitant Variation: A phenomenon that varies in any manner (whether directly or inversely) whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon (or is somehow causally connected with it).

> In some cases of cause and effect, it is not a question of all or nothing, but of more or less. Let's go back to the *sauce piquante*. Suppose Sue is a picky eater – she ate very little *sauce piquante* and only got slightly sick. By contrast, Harold went back to the buffet line for a second helping, and he got the sickest of all.

We need to recognize that the methods, while helpful, are by no means fool-proof.

- All of the methods assume that the person using the methods has an independent sense of causation. For example, they do not show which of two correlated phenomena is cause and which is effect. Perhaps lung cancer causes smoking.
- The methods will not work unless the cause is among the phenomena under investigation. For example, if I am trying to determine the cause of recurring indigestion, I will probably focus on food. But maybe my indigestion is caused by stress or some other emotional factor. If I only consider food, I will not find the cause.

- If there is more than one common ingredient in all the phenomena under investigation, I may mistakenly choose the wrong one. Recall the case of the scientific drinker. He attempted to determine the cause of his frequent inebriation by carefully noting what he drank. He looked back over his list at the end of the week. Monday night: vodka and orange juice. Tuesday night: gin and orange juice. Wednesday night: bourbon and orange juice. Thursday night: Scotch and orange juice. Friday night: rum and orange juice. By Saturday he was convinced that his inebriation was caused by orange juice.
- Sometimes two things that are statistically correlated are not causally connected to each other, but to a third thing. For example, my health has declined in proportion to my decrease in smoking. Should I therefore conclude that the decrease in smoking has caused the decline in health? I don't think so. I think both are age-related. I have experienced a natural decline in health as I have grown older, and I have also reduced my consumption of many things, tobacco among them, during the same time period.

Chapter 13 – Scientific Method

The Hypothetico-Deductive Technique

Although there is no way to mechanize the creative aspect of scientific theories, we can state the general steps involved in the formulation and testing of a theory. I will use the theory of evolution as an example.

- First, a tentative hypothesis is formulated. How and why it is formulated is irrelevant. It may have been a stroke of genius. Or, like the theory of evolution, it may have been around for centuries before someone figured out how to turn it from fantasy into science.
- Then additional evidence is collected. Darwin did this in his famous voyage of discovery.
- Next, the hypothesis is revised as necessary. Darwin did not understand the mechanism of evolution – how it actually worked – until he discovered the various species of finches on the Galapagos Islands.
- Then comes deducing further consequences. This takes the form of a prediction: if this hypothesis is correct, we should be able to observe so-and-so. Darwin said that if humans descended from apes, as his theory suggested, then we should find fossil evidence of intermediate species.
- Finally, test the consequences: The predictions can be confirmed or disconfirmed by observation and experiment. And in fact fossil evidence continues to be discovered that appears to fill in the gap between ape and man, although not in the way that Darwin envisioned it..